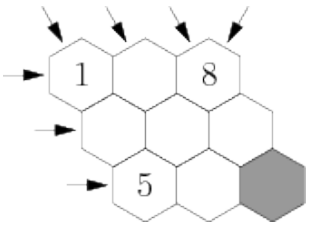
San Diego Math League High School Division, Round 2A
January 31, 2009

1. The digits 1 through 9 are placed in the hexagonal array so that all seven lines of 3 adjacent hexagons have the same sum. Each digit is used once. What number goes in the shaded hexagon?



(A) 2 (B) 3 (C) 4 (D) 6 (E) 7

2. The triplet (x, y, z) of positive real numbers satisfies the system

$$\begin{aligned} x + y + xy &= 19, \\ y + z + yz &= 11, \\ z + x + zx &= 14. \end{aligned}$$

What is $x + y + z$?

(A) 3 (B) 5 (C) 6 (D) 9 (E) 11

3. Let A_n be the sum of the first n terms of a particular arithmetic series, and let B_n be the sum of the first n terms of another arithmetic series. If

$$\frac{A_n}{B_n} = \frac{7n+1}{4n+27}$$

for all $n \geq 2$, then the ratio of the 11^{th} term of the first series to the 11^{th} term of the second series is

(A) 4/3 (B) 3/2 (C) 7/4 (D) 78/71 (E) 27/7

4. The pages of a book are numbered 1 through n. When the page numbers of the book were added, one of the page numbers was mistakenly added twice, resulting in an incorrect sum of 2009. What was the number of the page that was added twice?

(A) 54 (B) 55 (C) 56 (D) 57 (E) 58

5. What is the value of the sum

$$\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \frac{1}{7 \cdot 9} + \cdots + \frac{1}{199 \cdot 201} \ ?$$

(A) 2/5 (B) 100/201 (C) 50/101 (D) 3/7 (E) 1/2

6. In a triangle with side-lengths a, b, c we have

$$(a + b + c)(a + b - c) = 3ab.$$

Then the measure of the angle opposite to the side of length c is

(A) 15° (B) 30° (C) 135° (D) 60° (E) 120°

7. If $a > 1$, $x > 0$ and
$$(2x)^{\log_a 2} - (3x)^{\log_a 3} = 0,$$
then x is
(A) $\frac{1}{216}$ (B) $\frac{1}{6}$ (C) 1 (D) 6 (E) 12

8. Consider those functions f that satisfy $f(x+5) + f(x-5) = f(x)$ for all real x. Any such function is periodic, and there is a least common positive period p for all of them. Find p.
(A) 10 (B) 15 (C) 20 (D) 25 (E) 30

9. Dave buys a lottery ticket, which requires that he pick six different integers from 1 through 46, inclusive. He chooses his numbers so that the sum of the base-ten logarithms of his six numbers is an integer. It so happens that the integers on the winning ticket have the same property: the sum of the base-ten logarithms is an integer. What is the probability that Dave holds the winning ticket?
(A) 1 (B) 1/2 (C) 1/3 (D) 1/4 (E) 1/5

10. The triangle ABC is a right triangle with $\angle A = 90°$. Points M and N are the midpoints of the sides BC and CA respectively. The median AM is perpendicular on the median BN, and the side BC has length 1. Then the length of BN is
(A) $\sqrt{2}$ (B) $\frac{1}{2\sqrt{2}}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$ (E) $\frac{\sqrt{2}}{2}$

11. The number
$$\frac{(2^{30}-1)(2^5-1)(2^3-1)(2^2-1)}{(2^{15}-1)(2^{10}-1)(2^6-1)}$$
equals
(A) 127 (B) 255 (C) 295 (D) 331 (E) 511

12. A class of 10 students took a math test. Each problem was solved by exactly 7 of the students. If the first nine students each solved 4 problems, how many problems did the tenth student solve?
(A) 6 (B) 2 (C) 4 (D) 7 (E) 5

13. A tetrahedron has all its faces triangles with sides 13, 14 and 15. Its volume can be written as $m\sqrt{n}$, where m, n are positive integers, and n is square-free (that is, there is no prime number p such that $p^2 \mid n$.) Then $m + n$ equals
(A) 85 (B) 94 (C) 97 (D) 124 (E) 133

14. A committee of 5 is to be chosen from a group of 10 people. How many ways can it be chosen, if Dave and Richard must serve together or not at all, and Tina and Val refuse to serve with each other?
(A) 45 (B) 56 (C) 74 (D) 80 (E) 86

15. A point A is 9 units from the center O of a circle of radius 15. How many different chords of the circle pass through A and have integer lengths?
(A) 0 (B) 2 (C) 6 (D) 12 (E) an infinity

San Diego Math League High School Division, Round 2B
January 31, 2009

1. Find all triples of real numbers (a, b, c) such that $2a^2 - 4b^2 - 5bc = 24$ and $a : b : c = 6 : 2 : 5$.

2. Three married couples arrange a party. All six people arrive at different times to the party. Each person, upon arriving, shakes the hand of everyone already there, except his or her own spouse. When everyone has arrived, Bob, one of the husbands, asks all the others how many hands they shook upon arriving, and gets five different answers. How many hands did Bob shake upon arriving?

3. Calculate the sum of all divisors of the form $2^x \cdot 3^y$ (with $x, y > 0$) of the number $N = 19^{88} - 1$.

4. Find the smallest positive multiple n of 15 such that every digit of n is either 0 or 7.

5. In the trapezoid $ABCD$, with $AB \parallel CD$ and $AB > CD$. Let N be a point on the side BC, and let P be the intersection between the lines DB and AN. If $\dfrac{NA}{NP} = 2\dfrac{DB}{PB}$, what is $\dfrac{AB}{CD}$?

6. If ABC is a triangle such that $\max\{\angle A, \angle B\} = \angle C + 30^\circ$, and the ratio of the radius of the circumscribed circle to the radius of the inscribed circle is $\sqrt{3} + 1$, then find the measure of $\angle C$.

7. The distinct real numbers x, y satisfy $x^2 = 33y + 907$ and $y^2 = 33x + 907$. Find xy.

8. The four-digit numbers that do not contain the digit 0 are listed in increasing order. Find the last three digits of the 2009^{th} number.